LOCALIZATION OF NON-ABELIAN FIELD IN FIVE-DIMENSIONAL BRANEWORLD MODELS

Dewi Wulandari[1, 2], Triyanta[2]
[1]Physics Department, Universitas Negeri Medan, Jalan Willem Iskandar, Pasar V, Medan Estate 20221, Indonesia.
email : wulandaridewi@unimed.ac.id
[2]Theoretical High Energy Physics and Instrumentation Research Division, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Jalan Ganesha 10 Bandung 40132, Indonesia.
email : triyanta@fi.itb.ac.id

Abstrack, ***We study the localization properties of non-Abelian field in the representation of SU(2) group both in the modified Randall-Sundrum (MRS) and in the original Randall-Sundrum (RS) models. We investigate the conditions of localization for each model. We derive the equation of motion for the field. We solve the field equation corresponding to the extra dimension and analyze the localization properties of the field on the brane. We obtain that the MRS model has different field localization properties compared to the RS model. We find that non-Abelian field is localizable on the brane of the MRS model with specific conditions but the field is not localizable on the brane of the RS model.***

Keywords:
extra dimension, field localization, 5D braneworld, non-Abelian field,

INTRODUCTION

The gauge field theory becomes the basic theory of elementary particles. For example, the quantum electrodynamics theory which is a particular case of gauge theory completely confirmed by experiments. Another example is the weak interaction model, four-fermion interaction which has been introduced by interaction with vector particles, non-Abelian field (Yang-Mills field). The existing experiment data along with the requirement of gauge invariance led to the prediction of weak neutral currents and of a new quantum number for hadrons (charm).

Non-abelian field has an essential place in theoretical physics to describe theory of elementary particles. Chen Ning Yang and Robert Mills generalized the principle of gauge invariance of the interaction of electric charges for the case of interacting isospin. They introduced a vector field known as non-Abelian field (Yang-Mills field) which need recall some notations from the theory of compact Lie groups. This a compact group which has no invariant commutative (Abelian) sub groups. The number of independent parameters which characterize an arbitrary element of the group (that is dimension) is equal to *n*. The representation of *n x n* matrices (adjoint representation) have been introduced among the representations of this group and its Lie algebra. The simplest example of such group is the SU(2) group. The dimension of this group is *3*, the Lie algebra in adjoint representation is given by the antisymmetric *3 x 3* matrices as generators of the matrices which can be chosen as follows

$$T^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, T^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, T^3 = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \qquad (1)$$

The generator T^a can be normalized by condition

$$tr(T^a T^b) = -2\delta^{ab}. \qquad (2)$$

In this case the structure constant f^{abc} which take part in the condition

$$\left[T^a, T^b\right] = f^{abc}T^c \qquad (3)$$

are completely antisymmetric.

The vector field $w_\mu(x)$ is given in the adjoint representation of the Lie algebra which is defined by its coefficients $w^a_\mu(x)$ corresponding to the base of the generators T^a

$$w_\mu(x) = w^a_\mu(x)T^a \qquad (4)$$

w_μ is the gauge potential analogues to the electromagnetic field a_μ in the case of the group U(1) which is commutative (Abelian group). w_μ is a vector in the internal space while a_μ is analogues as object (only had one component) (Ryder, 1984). The U(1) abelian group is generalization of the non-Abelian group.

The braneworld framework describe that the matter fields (fundamental fields) and gauge interaction in the standard model are localized on a 3+1-dimensional hypersurface called brane. The brane is confined in a higher-dimensional spacetime known as bulk while gravity propagates along the bulk. Localization of matter fields on a four-dimensional brane explains why low energy physics is effectively in four-dimensional spacetime, and why gravity is the weakest among four fundamental interaction fields. The localization properties of fundamental fields in the MRS model are much better as compared to the RS model (P. Jones et.al, 2013; Triyanta et.al, 2013, D. Wulandari et.al, 2017). The localization of fundamental fields in the RS model is still unresolved issue. This model is not a perfect braneworld model in terms of field localization since not all types of fundamental matter fields are localized on 3+1 brane in a simple manner (M. Gogberashvili, 2002; L. Randall and R. Sundrum, 1999; B. Bajc and G. Gabadadze, 2000). In fact, only massless scalar fields are localizable on the brane for decreasing warp factor. On the other hand for increasing warp factors, only massless spinor fields are localizable on the brane. Meanwhile neither decreasing nor increasing warp factor could localize the vector filed on the brane of RS model. This fact led authors in Ref. P. Jones et.al, (2013) introduce the MRS model specified by the metric

$$ds^2 = a^2(x^5)\left[\eta_{\mu\nu}dx^\mu dx^\nu - dx^5 dx^5\right] \qquad (5)$$

with $a = e^{-k|x^5|}$ is warp factor with k is a constant. One could choose warp parameter i.e. $k>0$ corresponds to the decreasing warp factor while $k<0$ corresponds to the increasing warp factor. $x^5 = r$ is the fifth coordinate, and $\eta_{\mu\nu}$ is the Minkowski metric with signature (+,-,-,-) whereas the RS model (L. Randall and R. Sundrum, 1999) is characterized by a metric of the form

$$ds^2 = a^2(x^5)\eta_{\mu\nu}dx^\mu dx^\nu - dx^5 dx^5 \qquad (6)$$

To represent the fifth coordinate in the RS model we use *y*-coordinate instead of *r*-coordinate.

LOCALIZATION OF NON-ABELIAN FIELD IN THE MRS MODEL

The localization of non-Abelian field both in the RS and in the MRS models has been analyzed in Ref. D. Wulandari et.al, (2017). The representation of the non-Abelian field in Ref. D.Wulandari et.al, (2017) is given in general form SU(n). In this section we will investigate the localization of non-Abelian field in the representation of vector form SU(2). The action describing the interaction of Yang-Mills field is defined as follows

$$S=\int d^5x\left[-\sqrt{g}\,\frac{1}{4}\vec{W}_{AB}\vec{W}^{AB}\right], \qquad (7)$$

where $\vec{W}_{AB}=\partial_A\vec{W}_B-\partial_B\vec{W}_A+h\vec{W}_A x\vec{W}_B$ are the strengths of five-dimensional field tensor with is the five-dimensional vector field and *h* is a coupling constant. $\vec{W}_A$ is a vector in an internal symmetry space with n^2-1 dimension and its components are defined by $W_A^1,...,W_A^{n^2-1}$. The third term of five-dimensional electromagnetic field tensor shows that a vector field interacts with another vector field. By decomposing the five dimensional vector field to four-dimensional and extra-dimensional components, $\vec{W}_A=(\vec{W}_\alpha(x^5),\vec{W}_5)=(c(r)\vec{w}_\alpha(x^\beta),\vec{W}_5)$ and by choosing $\vec{W}_5$ is a constant so then the five-dimensional action separated into four-dimensional components and extra-dimensional components can be written as:

$$S=-\frac{1}{4}\int_0^\infty dx^5\sqrt{g}c^2g^{\alpha\nu}g^{\beta\sigma}\int d^4x\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dx^5\sqrt{g}c^3g^{\alpha\nu}g^{\beta\sigma}\int d^4xh\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)-\frac{1}{4}\int_0^\infty dx^5\sqrt{g}c^3g^{\alpha\nu}g^{\beta\sigma}\int d^4xh\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dx^5\sqrt{g}c^4g^{\alpha\nu}g^{\beta\sigma}\int d^4xh^2\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\vec{w}_\nu\times\vec{w}_\sigma\right)$$
$$-\frac{1}{2}\int_0^\infty dx^5\sqrt{g}\left(\partial_5c\right)^2g^{\alpha\nu}g^{55}\int d^4x\vec{w}_\alpha\cdot\vec{w}_\nu$$
$$-\frac{1}{2}\int_0^\infty dx^5\sqrt{g}c^2g^{\alpha\nu}g^{55}\int d^4x\left(\vec{w}_\alpha\times\vec{W}_5\right)\cdot\left(\vec{w}_\nu\times\vec{W}_5\right) \qquad (8a)$$

Recalling the MRS metric (5), $\sqrt{g}=a^5$, $g^{\mu\nu}=a^{-2}\eta^{\mu\nu}$, $g^{55}=-a^{-2}$ and integrating the action over the extra dimension from 0 to ∞, the above equation can be written as follows

$$S=-\frac{1}{4}\int_0^\infty dra(r)c^2(r)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dra(r)c^3(r)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)-\frac{1}{4}\int_0^\infty dra(r)c^3(r)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dra(r)c^4(r)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h^2\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\vec{w}_\nu\times\vec{w}_\sigma\right)$$
$$+\frac{1}{2}\int_0^\infty dra(r)\left(\partial_rc\right)^2\int d^4x\,\eta^{\alpha\nu}\vec{w}_\alpha\cdot\vec{w}_\nu$$
$$+\frac{1}{2}\int_0^\infty dra(r)c^2(r)\int d^4x\eta^{\alpha\nu}\left(\vec{w}_\alpha\times\vec{W}_5\right)\cdot\left(\vec{w}_\nu\times\vec{W}_5\right) \qquad (8b)$$

The vector field can be localized on the brane if the integrals over the extra dimension from 0 to ∞ are finite and then the localization conditions read

$$\int_0^\infty drac^2=N_1;\ \int_0^\infty drac^3=N_2;\ \int_0^\infty drac^4=N_3, \qquad (9a)$$

$$\frac{1}{2}\int_0^\infty dra(r)\left(\partial_rc\right)^2=\frac{1}{2}m_w{}^2, \qquad (9b)$$

$$\vec{W}_5=\vec{W}_r=0. \qquad (9c)$$

Note taking $N_1 = N_2 = N_3 = 1$, finally, the five dimensional action in eq. (8b) is analogue with the case of four-dimensional action in Minkowski spacetime

$$S = \int d^4x \left[\frac{1}{2} m_w{}^2 \eta^{\alpha\nu} \vec{w}_\alpha \cdot \vec{w}_\nu - \frac{1}{4} \eta^{\alpha\nu} \eta^{\beta\sigma} \left(\partial_\alpha \vec{w}_\beta - \partial_\beta \vec{w}_\alpha \right) \cdot \left(\partial_\nu \vec{w}_\sigma - \partial_\sigma \vec{w}_\nu \right) \right]$$
$$- \frac{1}{4} \int d^4x \eta^{\alpha\nu} \eta^{\beta\sigma} h \left(\partial_\alpha \vec{w}_\beta - \partial_\beta \vec{w}_\alpha \right) \cdot (\vec{w}_\nu \times \vec{w}_\sigma)$$
$$- \frac{1}{4} \int d^4x \eta^{\alpha\nu} \eta^{\beta\sigma} h (\vec{w}_\alpha \times \vec{w}_\beta) \cdot \left(\partial_\nu \vec{w}_\sigma - \partial_\sigma \vec{w}_\nu \right)$$
$$- \frac{1}{4} \int d^4x \eta^{\alpha\nu} \eta^{\beta\sigma} h^2 (\vec{w}_\alpha \times \vec{w}_\beta) \cdot (\vec{w}_\nu \times \vec{w}_\sigma). \quad (10)$$

From localization condition eq. (9a) the condition can be fulfilled for $c(r) = 1$ and it will give impact on the condition eq. (9b) which gives massless vector fields, $m_w = 0$. Also, from the localization condition eq. (9a) since $c(r) = 1$ gives $\int_0^\infty dr a(r) = 1$. It means that the system can be localized for decreasing warp factor (*k=1*).

The equation of motion for vector fields interacting with other vector fields can be derived by using Euler-Lagrange equation corresponding to the action (7)

$$\partial_P \left(\sqrt{g} \vec{W}^{PQ} \right) + \sqrt{g} h \vec{W}_P \times \vec{W}^{QP} = 0. \quad (11)$$

Recalling $\vec{W}_{PQ} = \partial_P \vec{W}_Q - \partial_Q \vec{W}_P + h \vec{W}_P x \vec{W}_Q$ and $\vec{W}_P = (\vec{W}_\alpha(x^5), \vec{W}_5) = (c(r)\vec{w}_\alpha(x^\beta), \vec{W}_5)$, with $\vec{W}_5 = const$, so then the equation of motion (11) becomes:

$$\partial_\mu \left(\sqrt{g} g^{\mu\nu} g^{Q\alpha} \right) \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$
$$\sqrt{g} g^{\mu\nu} g^{Q\alpha} \partial_\mu \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$
$$\partial_r \left(\sqrt{g} g^{55} g^{Q\alpha} \right) \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right)$$
$$\sqrt{g} g^{55} g^{Q\alpha} \partial_r \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right)$$
$$- \sqrt{g} g^{\mu\nu} g^{Q\alpha} h c^2(r) \vec{w}_\mu \times \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) + h c(r)(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$
$$- \sqrt{g} g^{55} g^{Q\alpha} h \vec{W}_5 \times \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right) = 0$$

(12)

Based on eq. (12), the equation of motion for vector field on the bulk (*Q=5*)

$$\partial_\mu \left(\sqrt{g} g^{\mu\nu} g^{5\alpha} \right) \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$
$$\sqrt{g} g^{\mu\nu} g^{5\alpha} \partial_\mu \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$
$$\partial_r \left(\sqrt{g} g^{55} g^{5\alpha} \right) \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right)$$
$$\sqrt{g} g^{55} g^{5\alpha} \partial_r \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right)$$
$$- \sqrt{g} g^{\mu\nu} g^{5\alpha} h c^2(r) \vec{w}_\mu \times \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) + h c(r)(r) \vec{w}_\nu \times \vec{w}_\alpha \right) - \sqrt{g} g^{55} g^{5\alpha} h \vec{W}_5 \times \left(\partial_r c(r) \vec{w}_\alpha + h c(r) \vec{W}_5 \times \vec{w}_\alpha \right) = 0$$

(13)

Since the MRS is a diagonal metric $(g^{5\alpha} = 0)$, the left hand-side of the above equation will vanish.

The equation of motion ony the brane $(Q = \beta = 0, 1, 2, 3)$ become:

$$\partial_\mu \left(\sqrt{g} g^{\mu\nu} g^{\beta\alpha} \right) \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right) \sqrt{g} g^{\mu\nu} g^{\beta\alpha} \partial_\mu \left(\left(\partial_\nu \vec{w}_\alpha - \partial_\alpha \vec{w}_\nu \right) c(r) + h c^2(r) \vec{w}_\nu \times \vec{w}_\alpha \right)$$

$$\partial_r\left(\sqrt{g}g^{55}g^{\beta\alpha}\right)\left(\partial_r c(r)\vec{w}_\alpha + hc(r)\vec{W}_5\times\vec{w}_\alpha\right)$$
$$\sqrt{g}g^{55}g^{\beta\alpha}\partial_r\left(\partial_r c(r)\vec{w}_\alpha + hc(r)\vec{W}_5\times\vec{w}_\alpha\right)$$
$$-\sqrt{g}g^{\mu\nu}g^{\beta\alpha}hc^2(r)\vec{w}_\mu\times\left(\left(\partial_\nu\vec{w}_\alpha-\partial_\alpha\vec{w}_\nu\right)+hc(r)(r)\vec{w}_\nu\times\vec{w}_\alpha\right)-\sqrt{g}g^{55}g^{\beta\alpha}h\vec{W}_5\times\left(\partial_r c(r)\vec{w}_\alpha+hc(r)\vec{W}_5\times\vec{w}_\alpha\right)=0$$
(14)

Recalling the MRS metric (5), $\sqrt{g}=a^5$, $g^{\mu\nu}g^{\beta\alpha}=a\eta^{\mu\nu}\eta^{\beta\alpha}$, $g^{55}g^{\beta\alpha}=-a\eta^{\beta\alpha}$ and using the localization condition $c(r)=1, \vec{W}_5=\vec{W}_r=0$ then the equation of motion for the vector field on the brane reduces into

$$\partial_\mu\vec{w}^{\mu\alpha}-h\vec{w}_\mu\times\vec{w}^{\mu\alpha}=0. \qquad (15)$$

The above equation is the same as the equation of motion for the vector field interacting with other vector field proposed in Yang-Mills theory in four-dimensional Minkowski spacetime with Lagrangian as follows

$$\mathrm{L}=-\frac{1}{4}\vec{w}_{\alpha\beta}\cdot\eta^{\alpha\sigma}\eta^{\beta\nu}\vec{w}_{\sigma\nu}. \qquad (16)$$

This fact also implies that the massless vector field (non-Abelian field) can be localized on the brane as long as $c(r)=1, \vec{W}_5=\vec{W}_r=0$ and warp factor should decrease (*k=1*).

LOCALIZATION OF NON-ABELIAN FIELD IN THE RS MODEL

The next investigating is the localization of non-Abelian field in the original five-dimensional RS model. The corresponding action is given in (7). Decomposing $\vec{W}_A=(\vec{W}_\alpha(x^5),\vec{W}_5)=(c(y)\vec{w}_\alpha(x^\beta),\vec{W}_5)$ and choosing $\vec{W}_5$ a constant, the five-dimensional action separated into four-dimensional and extra-dimensional components in the RS model can be written as:

$$S=-\frac{1}{4}\int_0^\infty dyc^2(y)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dyc^3(y)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h\left(\partial_\alpha\vec{w}_\beta-\partial_\beta\vec{w}_\alpha\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dyc^3(y)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\partial_\nu\vec{w}_\sigma-\partial_\sigma\vec{w}_\nu\right)$$
$$-\frac{1}{4}\int_0^\infty dyc^4(y)\int d^4x\eta^{\alpha\nu}\eta^{\beta\sigma}h^2\left(\vec{w}_\alpha\times\vec{w}_\beta\right)\cdot\left(\vec{w}_\nu\times\vec{w}_\sigma\right)$$
$$+\frac{1}{2}\int_0^\infty dya^2(y)(\partial_y c(y))^2\int d^4x\eta^{\alpha\nu}\vec{w}_\alpha\cdot\vec{w}_\nu$$
$$+\frac{1}{2}\int_0^\infty dya^2(y)c^2(y)\int d^4x\eta^{\alpha\nu}\left(\vec{w}_\alpha\times\vec{W}_5\right)\cdot\left(\vec{w}_\nu\times\vec{W}_5\right)(17)$$

The Vector field can be localized on the brane if the action integrals over extra dimension from $y=0$ to $y=\infty$ are finite giving the following localization conditions

$$\int_0^\infty dyc^2(y)=N_1;\int_0^\infty dyc^3(y)=N_2;\int_0^\infty dyc^4(y)=N_3,(18a)$$

$$\frac{1}{2}\int_0^\infty dy a^2(y)\left(\partial_y c(y)\right)^2 = \frac{1}{2}m_w^{\ 2}, \qquad (18b)$$

$$\vec{W}_5 = \vec{W}_y = 0. \qquad (18c)$$

Note choosing $N_1 = N_2 = N_3 = 1$, finally the five dimensional action in eq. (17) is analogue with the case of four-dimensional action in Minkowski spacetime in eq. (10).

Note taking $N_1 = N_2 = N_3 = 1$ from the localization condition eq. (18a), those equations can be satisfied for $c(y) = 1$. The consequence of this condition contributes a massless vector field, $m_w = 0$ in (18b). For $c(y) = 1$ the integrals over the fifth coordinate in condition (18a) are infinite indicating that non-Abelian field is not localizable on the brane.

CONCLUSIONS

We analyzed the localization properties of non-Abelian field both in the MRS and in the original RS models. These metrics that were introduced in Ref. P. Jones et.al (2013) are conformally flat. Geometrically, the modified model is quite distinct from the original model representing those metrics have different spacetime. The expressions of Ricci scalars and Einstein tensors for both models (P. Jones et.al, 2013) indicate that both models have different spacetime. To start investigating of the localization properties of the vector field we derived the conditions for localization. These conditions can be found by decomposing the five-dimensional action of non-Abelian field into the extra-dimensional and the four dimensional parts. The fields are localizable on the brane if action integrals over the extra dimension from $x^5 = 0$ to $x^5 = \infty$ are finite. Note taking $N_1 = N_2 = N_3 = 1$, the investigating of localization conditions in the MRS model (9a) need $c(r) = 1$ giving decreasing warp factor to localize the field and massless vector field (9b). Finally taking $c(r) = 1$, the equation of motion for vector field on the brane (14) reduce into the equation of motion in Minkowski spacetime (15) indicating the vector field is localizable on the brane. On the other hand in the RS model, the condition $N_1 = N_2 = N_3 = 1$ in (18a) gives $c(y) = 1$ contributing the massless vector field (18b). However taking $c(y) = 1$ gives the integrals of localization condition (18a) are infinite thus the vector field is not localizable in the RS model.

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