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MARKOV CHAIN TO CONTROL GENERIC DRUG IN INVENTORY PROBLEM

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Abstract: Have the generic drugs inventory been optimal, neither excessive nor deficient? Inadequate inventory can cause in losses due to increased ordering and storage costs. Shortage of generic drug will cause a decrease of public health services. Given demand generic drugs which always fluctuates, so stochastic process is the right method. In this study, the optimal level of generic drugs inventory will be analyzed by using the Markov chain. Steady state conditions were achieved when the same results obtained with the previous iteration result. There are 7 linear equations produced, each accompanied by 7 levels of inventory, optimal order quantities and also the costs.

Keywords: Demand; Generic Drug; Inventory; Markov Chain.

Abstrak: Apakah persediaan obat generik sudah optimal, tidak berlebih maupun kekurangan? Persediaan yang tidak optimal dapat mengakibatkan kerugian akibat bertambahnya biaya pemesanan maupun biaya penyimpanan. Kekurangan persediaan obat generik bahkan juga akan mengakibatkan turunnya tingkat layanan kesehatan masyarakat. Mengingat tingkat permintaan obat generik yang berfluktuasi maka proses stokastik menjadi pilihan metode yang tepat. Pada penelitian ini, tingkat persediaan obat generik yang optimal akan dianalisis dengan menggunakan markov chain. Kondisi steady state tercapai ketika hasil yang sama diperoleh dengan hasil dari iterasi sebelumnya. Ada 7 persamaan linier yang dihasilkan, masing-masing disertai dengan 7 tingkat persediaan dan besaran pemesanan optimal dan juga besaran biaya.

Kata kunci: Permintaan, Obat Generik; Persediaan; Rantai Markov.

INTRODUCTION

The need for generic drugs is very high , in connection of BPJS program which provided for all levels of society. Generic drugs inventory have become issue should get full attention from the government. Good Health care services must be supported by the availability of generic drugs optimally. Shortage of generic drugs inventory directly will cause decline the level of service community health.

The inventory is important factor but it was always failed to control. Controlling inventory aims at determining the scale of inventory by taking into the costs.Thus the inventory problems have a direct effect on level of service. Not only lack inventory causes problems but also big inventory increases in interest expenses, the storage cost and maintenance in warehouse and also depreciation and the quality which can’t be maintained or expired date.

It is needed a management inventory to analyze the level of inventory in most optimum position. Base data is necessary to fulfill how any period of in the future, with exact inventory is not

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too much or a little. In addition, there are some costs to be counted on such the production cost, storage cost, and demand inventory cost. Then planning inventory needs a forecasting method. Simple forecasting technique can be used by counting on the last request to estimate the future needs.

The problems of generic drugs inventory becomes more complicated because the demand of generic drugs is not settled, uncertain characteristics and inclined to fluctuate from time to time (Arsky, R.B. & De Long J.B, 1993). The probability characteristic of markov stochastic chain becomes one the right choice to overcome the uncertainty. By using a stochastic process, we can predict the supplies amount to be made for whatever the cause of uncertainty factors as the forecasting needs, stock, the number of safety inventory, and the uncertainty derived from suppliers for example the quantity of generic drug from initial agreement.

A stochastic process is defined as drafting process and indexing a random variables $\{X_t\}$, with an index t be on a set of t. Sometimes t is assumed as non negative integer, and Xt presents on measurable characteristics that we pay attention at t. as an example, Xt presents on the level of generic drugs inventory in the end of month. The decision stochastic process can be explained by a number of limited state. The transitional probability between the states was explained by using markov chain. The structure of costly process decision was also described by a matrix whose the elements states that income or fare resulted of the movement from one state to another.

Markov Chain is a series of process events where the conditional opportunity for future events depends on the current event and it does not depend on the previous situation (Chan, 2012). This technique is used to assist in predicting changes that might occur in the future. Where the number of steps in which a state is reachable or accessible from another state in a finite Markov chain with M (≥ 2) states. Markov Chain are widely used models in a variety of areas of theoretical and applied mathematics and science, including statistics, operations research, industrial engineering, linguistic, artificial intelligence, demographics, genomics (Von Hilgers, & Langville, A, 2006)

Some previous study a novel dividend valuation model is put forward by using a Markov Chain (Luca L.G., & Carlo, 2003). Barbu et all propose further advancements in the Markov chain stock model. Markov decision process approach, the states of a Markov chain represent possible states of demand for the inventory item. Meanwhile, the decision problem using Markov chain combined with demand and inventory positions and some cost such as the production cost, holding cost, shortage cost and sales price are to generate the profit (Kizito, 2015).

The problems discussed in this research was a controlling inventory problem using a stochastic process markov chain (Barbu, 2017; Kizito, 2015; Lakdere, 2001). It is hoped the result of this research can determine optimal inventory, by considering various cost factors. This article were arranged in four parts, where the section 2 explains the basic idea about the characteristic of stochastic process and forecasting by using markov chain. Then section 3 describes the result of the study with 7 linear equations completely using a solution and the process of iterasi. The last section 4 is conclusion, it provides an optimum choice inventory.

METHODOLOGY

Markov Chain is a series of process events where the conditional opportunity for future events depends on the current event and it does not depend on the previous situation (Chan, 2012). A Stochastic Process X_t is called by Markov if:

$$P\{X_{t+1} = j | X_0 = k_0, X_1 = k_1, \cdots, k_{t-1} = X_{t-1}, X_t = i\}$$

$$P\{X_{t+1} = j | X_t = i\}$$

where

$$t = 0,1,2,\cdots,n$$

in other words, Markov characteristic states that conditional probability of next events with past events and current events $X_t = i$ is independent to past events and it depends on current events. So stochastic process $\{X_t, t = 0,1,2,\cdots,n\}$ is Markov Chain because that process has Markov characteristic. Conditional probabilities $P\{X_{t+1} = j | X_t = i\}$ for Markov chain is called by transition probability (move one step) if *i* and *j* are applied:

$$P\{X_{t+1} = j | X_t = i\} = P\{X_t = j | X_0 = i\}$$

for $t = 0,1,2,\cdots,n$

$$P_{ij}^n \geq 0 \; untuk\; semua\; i\; dan\; j, n = 0,1,2,\cdots.$$

and

$$\sum_{j=0}^{\infty} P_{ij}^{(n)} = 1 \; untuk\; semua\; i; n = 0,1,2,\cdots$$

Conditional probability given a notation $P_{ij}^{(n)}$ is known as transition probability *n* step, it is also called by conditional opportunities from random variable *x*, by starting in condition level *j* after *n* step.

Because P_{ij}^n is conditional probabilities, that probabilities must be non-negatif, and the process must change to another *state*.

Then it could be formed a transition opportunity matrix because of stochastic process $\{X_t, t = 0,1,2,\cdots,n\}$ is Markov Chain with a finite set $\{0,1,2,\cdots,n\}$. Transition opportunity matrix (one step) from $\{X_t, t = 0,1,2,\cdots,n\}$, is noted with **P** is a matrix with element *(i,j)* is P_{ij}.

$$\boldsymbol{P} = \begin{bmatrix} P_{00} & P_{01} & P_{02} & \cdots & P_{0n} \\ P_{10} & P_{11} & P_{12} & \cdots & P_{1n} \\ P_{20} & P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{n0} & P_{n1} & P_{n2} & \cdots & P_{nn} \end{bmatrix}$$

If $P_{ij} \geq 0$ dan $\sum_{j=0}^{\infty} P_{ij}^{(n)} = 1; i,j = 0,1,2,\cdots,n.$

For transition opportunitie n-step is used Chapman-Kolmogorof equation:

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^{(n)} P_{kj}^{(m)}, n, m \geq 0; i, j \geq 0$$

It can be shown that $\mathbf{P}^{(n)} = \mathbf{P}^n$ where transition probability matrix *n* step $\mathbf{P}^n$ is transition probability matrix one step quadrate *n*.

for *n* = 1 and *m* = *n* - 1 so the equation is:

$$P_{ij}^{(n)} = \sum_{k=0}^{\infty} P_{ik} P_{kj}^{(n-1)}$$

$$= \sum_{k=0}^{\infty} P_{ik}^{(n-1)} \sum_{k=0}^{\infty} P_{kj}$$

For *n* the step obtained is:

$$\mathbf{P}^{(n)} = \mathbf{P}\mathbf{P}^{(n-1)} = \mathbf{P}^{(n-1)}\mathbf{P}$$

$$= \mathbf{P}\mathbf{P}^{n-1} = \mathbf{P}^{n-1}\mathbf{P}$$

$$= \mathbf{P}^n$$

opportunity *State* n the step in Markov Chain process is defined as row vector

$$p^n = \left(p_1^{(n)}, p_2^{(n)}, \cdots\right), n = 1, 2, \cdots$$

Is opportunity vector *state* after *n* step $p_j^{(n)}$ is opportunity vector in *state j* after *n* step, where $n \geq 1, j \geq 0$.

$$p_j^{(n)} = p(x_n = j)$$

$$\sum_{i=0}^{\infty} p(x_n = j, x_0 = i)$$

$$= \sum_{i=0}^{\infty} p(x_0 = i) p(x_n = j | x_0 = i)$$

$$= \sum_{i=0}^{\infty} p_i^0 p_{ij}^n$$

Iteration stops when a steady state opportunity is obtained which is a transition opportunity that has reached a balanced or fixed condition.

$$\boldsymbol{P}^m = \begin{bmatrix} P_{00}^m & P_{01}^m & P_{02}^m & \cdots & P_{0n}^m \\ P_{10}^m & P_{11}^m & P_{12}^m & \cdots & P_{1n}^m \\ P_{20}^m & P_{21}^m & P_{22}^m & \cdots & P_{23}^m \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{n0}^m & P_{n1}^m & P_{n2}^m & \cdots & P_{nn}^m \end{bmatrix}$$

where:

$$P_{ij}^m \geq 0$$

Results and Discussion Length

Data used in this research was generic drug data which was taken from Bina Kasih Medan Hospital, started on 1 January to 31 December 2018. Data was written every week, namely number of requests, storage cost in warehouse, and booking fee. Table 1 is distribution of requests that have been processed from the initial data by grouping in 7 classes of data.

Table 1 Distribution of Generic Drug Request.

Class	Request Data	Average Storage Cost (Rp)	Average Demand Cost (Rp)
1	26456825-41231071	1783000	363000
2	41231072-56005318	1800000	650000
3	56005319-70779565	1786000	757000

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4	70779586-85553832	1775000	528000
5	85553833-100328079	1808000	892000
6	100328080-115102326	1800000	1000000
7	115102327-129876573	1800000	940000
	Avarage	**1793000**	**733000**

From table 1 it can be seen that the avarage demand cost for generic drug in a year is Rp 733.000, and avarage storage cost for generic drug in a year is 1.793.000. By having seven level of initial inventory starts from state *i* is 0, then 14774246, and next 29548491, then 44322737, 59096983, and 73871229, state last is 88645474.

Based on markov chain rule that the decision making a state from a system has transition from state i to state $j = i + x - d$ with probability $P_{ij}(x) = P(d)$. Counting transition probability P_{ij}, Shortage cost E, Total cost C_i, for every alternative in every order. It was obtained the result in table 2 for first Row.

Table 2 First Row of Transition Probability, Shortage and Total Cost.

Value	**First Alternative Order**						
	41231071	**56005318**	**70779565**	**85553832**	**100328079**	**115102326**	**129876573**
$P_{0,0}$	1	0,875	0,771	0,479	0,396	0,146	0,104
$P_{0:14774246}$	0	0,125	0,104	0,292	0,083	0,25	0,042
$P_{0:29548592}$	0	0	0,125	0,104	0,292	0,083	0,25
$P_{0:44322734}$	0	0	0	0,125	0,104	0,292	0,083
$P_{i,59096983}$	0	0	0	0	0,125	0,104	0,292
$P_{0:73781229}$	0	0	0	0	0	0,125	0,104
$P_{0:88645474}$	0	0	0	0	0	0	0,125
E	30,008,617,189,461.	20,532,784,519,836	12,183,222,247,515	6,995,873,683,946	2,707,382,555,750	1,126,272,190,304	1793000
C_0	30,008,618,982,461	20,532,786,312,836	12,183,224,040,515	6,995,875,476,946	2,707,384,348,750	1,126,273,983,304	3586000

The iteration was continued for the initial inventory level of 14774246 which gives six alternative order levels. It ends on the seventh line with an initial inventory level of 88645474 which leaves only one alternative choice of order level namely ordering of 41231071 as described in table 3.

Table 3 Last Row (7th Row) Transition Probability, Shortage and Total Cost.

Value	**Alternative Demand Level 41231971**
$P_{88645474:0}$	0,104
$P_{88645474:14774246}$	0,042
$P_{88645474:29548592}$	0,25
$P_{88645474:44322734}$	0,083

$P_{88645474:59096983}$	0,292
$P_{88645474:73781229}$	0,104
$P_{88645474:88645474,}$	0,125
E	1,793,000.00
$C_0(x)$	64,977,136,028,000.00

Policy iteration is used to get optimal solution by determining first random policy (k = 0) $X = x_t^{(k)}$ and arranging probability matrix and the cost.

Choose first policy:

$$x_0^{(0)} = 129876573$$

$$x_{14744246}^{(0)} = 115102326$$

$$x_{29548492}^{(0)} = 100328079$$

$$x_{44322737}^{(0)} = 85553832$$

$$x_{59096983}^{(0)} = 70779565$$

$$x_{73871229}^{(0)} = 56005318$$

$$x_{88645474}^{(0)} = 41231071$$

From this policy it can be seen that if the initial inventory is at level 0 then the optimal alternative order is 129876573, and if initial inventory is in 14744246 so or optimal alternative order is 115102326, for initial inventory is 29548492, 44322737, 59096983, 73871229, 88645474 obtained optimal alternative order is 100328079, 85553832, 70779565, 56005318, 41231071.

The transition matrix is determined by taking the smallest optimum cost from the probability result obtained, so it gets a cost matrix:

$$\begin{bmatrix} 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \\ 0.104 & 0.042 & 0.25 & 0.083 & 0.292 & 0.104 & 0.125 \end{bmatrix} \begin{bmatrix} C_0 129876573 \\ C_{14774246} 115102326 \\ C_{29548492} 100328079 \\ C_{44322737} 85553832 \\ C_{59096983} 70779565 \\ C_{73871229} 56005318 \\ C_{88645474} 41231071 \end{bmatrix}$$

$$\begin{bmatrix} 3.586000 \\ 10{,}829{,}525{,}904{,}000 \\ 21{,}659{,}048{,}222{,}000 \\ 32{,}488{,}569{,}807{,}000 \\ 43{,}318{,}092{,}125{,}000 \\ 54{,}147{,}614{,}443{,}000 \\ 64{,}977{,}136{,}028{,}000 \end{bmatrix}$$

It was obtained seven linier equations based on initial policy that had been determined:

$$f_i^{(k)} = C_i\left(x_i^{(k)}\right) + \sum_j p_{ij}(x_i^{(k)}) f_j^{(k)}$$

That is:

$$f_0^{(0)} = 3.586000 + (0.104)f_0^{(0)} + (0.042)f_{14774246}^0 + (0.25)f_{29548492}^0 + (0.083)f_{44322737}^0 + (0.292)f_{59096983}^0 + (0{,}104)f_{73871229}^0 + (0{,}125)f_{88645474}^0$$

$$f_{14774246}^{\ \ 0} = 10{,}829{,}525{,}904{,}000 + (0.104)f_0^{(0)} + (0.042)f_{14774246}^0 + (0.25)f_{29548492}^0 + (0.083)f_{44322737}^0 + (0.292)f_{59096983}^0 + (0{,}104)f_{73871229}^0 + (0{,}125)f_{88645474}^0$$

$$f_{29548492}{}^{0} = 21{,}659{,}048{,}222{,}000 + (0.104)f_0{}^{(0)} + (0.042)f^0_{14774246} + (0.25)f^0_{29548492} + (0.083)f^0_{44322737} + (0.292)f^0_{59096983} + (0{,}104)f^0_{73871229} + (0{,}125)f^0_{88645474}$$

$$f_{44322737}{}^{0} = 32{,}488{,}569{,}807{,}000 + (0.104)f_0{}^{(0)} + (0.042)f^0_{14774246} + (0.25)f^0_{29548492} + (0.083)f^0_{44322737} + (0.292)f^0_{59096983} + (0{,}104)f^0_{73871229} + (0{,}125)f^0_{88645474}$$

$$f_{59096983}{}^{0} = 43{,}318{,}092{,}125{,}000 + (0.104)f_0{}^{(0)} + (0.042)f^0_{14774246} + (0.25)f^0_{29548492} + (0.083)f^0_{44322737} + (0.292)f^0_{59096983} + (0{,}104)f^0_{73871229} + (0{,}125)f^0_{88645474}$$

$$f_{73871229}{}^{0} = 54{,}147{,}614{,}443{,}000 + (0.104)f_0{}^{(0)} + (0.042)f^0_{14774246} + (0.25)f^0_{29548492} + (0.083)f^0_{44322737} + (0.292)f^0_{59096983} + (0{,}104)f^0_{73871229} + (0{,}125)f^0_{88645474}$$

$$f_{88645474}{}^{0} = 64{,}977{,}136{,}028{,}000 + (0.104)f_0{}^{(0)} + (0.042)f^0_{14774246} + (0.25)f^0_{29548492} + (0.083)f^0_{44322737} + (0.292)f^0_{59096983} + (0{,}104)f^0_{73871229} + (0{,}125)f^0_{88645474}$$

Linear equation was solved by the application of Lingo 10.

$f_0{}^{(0)} = 0$

$f_{14774246}{}^{0} = 0{,}3699808 \times 10^{14}$ =36,998,080,000,000

$f_{29548492}{}^{0} = 0$

$f_{44322737}{}^{0} = 0{,}5865712 \times 10^{14}$ = 58,657,120,000,000

$f_{59096983}{}^{0} = 0$

$f_{73871229}{}^{0} = 0{,}8031617 \times 10^{14}$ = 80,316,170,000,000

$f_{88645474}{}^{0} = 0{,}9114569 \times 10^{14}$ = 91,145,690,000,000

Iteration was continued by improving policy iteration to choose:

$$\min((C_i(x) + \sum_j p_{i,j}(x_t)f_j^{(k)})$$

where k + 1 new policy to get decision $x_i^{(k+1)}$, a new decision was obtained:

$x_0^{(1)} = 129876573$

$x_{14744246}^{(1)} = 115102326$

$x_{29548492}^{(1)} = 100328079$

$x_{44322737}^{(1)} = 85553832$

$x_{59096983}^{(1)} = 70779565$

$$x^{(1)}_{73871229} = 56005318$$

$$x^{(1)}_{88645474} = 41231071$$

The condition of steady state was obtained the result was the same with the previous result so the iteration was stopped. The results of this decision was optimal results with optimal costs.

$$C_0(129876573) = 3.586.000$$

$$C_0(115102326) = 10{,}829{,}525{,}904{,}000$$

$$C_0(100328079) = 21{,}659{,}048{,}222{,}000$$

$$C_0(85553832) = 32{,}488{,}569{,}807{,}000$$

$$C_0(70779565) = 43{,}318{,}092{,}125{,}000$$

$$C_0(56005318) = 54{,}147{,}614{,}443{,}000$$

$$C_0(41231071) = 64{,}977{,}136{,}028{,}000$$

CONCLUSION

With fluctuating generic drug demand level, resulted seven choices of optimal inventory levels based on seven early inventory assumptions:
Initial inventory 0 enable to maximum order is 129876573 with cost is Rp 3.586.000. Initial inventory 14774246, maximum order is 115102326 with cost is Rp 10,829,525,904,000. Initial inventory 29548492, maximum order is 1003280793 with cost is Rp 21,659,048,222,000. Initial inventory 44322737, maximum order is 85553832 with cst is Rp 32,488,569,807,000. Initial inventory 59096983, maximum order is 70779565 with cost is Rp 43,318,092,125,000. Initial inventory 73871229, maximum order is 56005318 with cost is Rp 54,147,614,443,000. Initial inventory 88645474, maximum order is 41231071 with cost is Rp 64,977,136,028,000.

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