Reflection transformation geometry assisted with Geogebra software

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Abstract *This study aims to discuss qualitative research methods literature study. The method used is literature study, data collection by searching for source and constructing from various sources such as books, journals and existing research. This study aims to help visualize and find concepts in the geometric transformation of reflections on points and planes that are reflected on O(0,0), on the X-axis, and on the Y-axis. The results of this study show that using Geogebra software can help visualize points and planes which is reflected about the point O(0,0), about the X, Y axes, the lines y = x, y = -x, x = h and x = k. By using Geogebra software, it can be seen that the results of point and plane reflections are reflected at point O(0,0), about the X, Y axes, the lines y = x, y = -x, x = h and x = k.* [REFLECTION TRANSFORMATION GEOMETRY ASSISTED WITH GEOGEBRA SOFTWARE] ***(J. Math. Nat. Sci., 2(2): 12 – 16 2022)***

Introduction

The process of learning mathematics is closely related to the mastery and application of concepts. The concepts in mathematics are abstract, sequential and have a relationship between one concept and another. Transformational geometry is one of the materials in learning mathematics that is important to understand and apply. Some of the things learned in Transformation Geometry include translations, rotations, reflections and dilations. Dinata (2019) states that the concept of transformational geometry follows mathematics as a deductive science. Every concept in transformation geometry begins with a definition or a postulate. Several theorems are derived from the given definitions. Furthermore, several problems can be solved with the theorem. Proving a theorem or solving a problem requires logical and deep thinking and requires good knowledge of algebra.

Considering that the concept of Transformation Geometry is quite broad in scope, the discussion in this study is focused on the reflection Transform Geometry contained in the Cartesian Coordinate System. The Cartesian coordinate system is used to determine each point in the plane by using two numbers commonly called the coordinate x (abscissa) and coordinate y (ordinate) of that point.
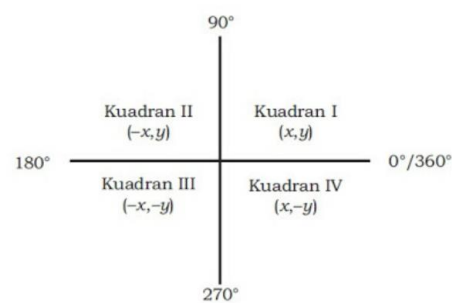

Figure 1. Cartesian coordinates.

Based on the picture above, there are 4 symmetrical planes bounded by the coordinate axes x and y where each plane is bounded by a plane called Quadrant. So that there are 4 Quadrants, namely:

- Kwadran I ($x > 0$, $y > 0$)
- Kwadran II ($x < 0$, $y > 0$)
- Kwadran III ($x < 0$, $y < 0$)
- Kwadran IV ($x > 0$, $y < 0$)

In this regard, the use of technology is of course very helpful in learning Transformational Geometry. According to Hanafi et al. (2017) that visualization aids play an important role in learning Transformational

Geometry. By using the data visualization that is displayed it can make it easier to understand the concept of Transformation Geometry. Therefore, there is a need for innovation in learning mathematics by using software that helps visualize Transformation Geometry, especially in reflection material.

One of the software that can support the learning of Transformational Geometry is Geogebra. Erlinawati (2018) stated that Geogebra has a simple appearance, the use of interactive software can make Geogebra software the right choice in conveying mathematical concepts. Geogebra as a learning medium displays material information in the form of demonstrations or visualizes mathematical concepts as well as being a tool for constructing these concepts. Likewise, according to Wulandari (2019) states that Geogebra is software for mathematics that integrates algebra, geometry, calculus and graphs of certain functions that can be used as a medium for learning mathematics to demonstrate and visualize mathematical concepts.

From the understanding related to Geogebra, it can be concluded that Geogebra can be used as a medium in learning mathematics, especially in Transformational Geometry. This is in line with Yuliardi's opinion (2013) that a number of studies show that GeoGebra can encourage students' discovery and experimentation processes in class. Geogebra is especially suitable for transformation geometry material because of the following reasons: (1) offers features that can transform objects on the screen, (2) allows users to draw geometric shapes easily and can measure precisely distances, angles, and broad, and (3) provides a click-drag feature that gives students the opportunity to describe or arrange geometric shapes in topics that require visualization such as geometric transformations in the reflection sub-chapter. It is hoped that Geogebra software users can help the learning process and have a positive impact on increasing interest in learning.

Materials and Methods

This research is a qualitative research using literature study method. The literature study method is a method of collecting data by obtaining information through books, magazines, newspapers, scientific journals and other literature that is relevant to the problem under study.

Results

Various kinds of reflection based on the mirror as follows

1. Reflection on the X-axis

If point P(x,y) is reflected about the X axis, then the image is obtained as follows

Mapping $P(x, y) \rightarrow P'(x', y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is called the matrix corresponding to the reflection about the X axis. So, it can be written:

$$M_x: P(x, y) \rightarrow P'(x', y') = P'(x, -y)$$

2. Reflection on the Y axis

If point P(x,y) is reflected about the Y axis, then the image is obtained as follows

Mapping $P(x, y) \rightarrow P'(x', y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ is called the matrix corresponding to the reflection about the Y axis. So, it can be written:

$$M_y: P(x, y) \rightarrow P'(x', y') = P'(-x, y)$$

3. Reflection on the O (0,0)

If the point P(x,y) is reflected to the origin O(0,0), then the image is obtained as follows:

Mapping $P(x, y) \rightarrow P'(x', y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is called the matrix corresponding to the reflection about the origin O (0,0). So, it can be written:

$$M_o: P(x, y) \rightarrow P'(x', y') = P'(-x, -y)$$

4. Reflection on the line y = x

If the point P(x,y) is reflected against the line y = x, then the image is obtained as follows:

Mapping $P(x, y) \rightarrow P'(x', y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is called the matrix corresponding to the reflection about the line y = x.

So, it can be written:

$$M_{y=x}: P(x,y) \rightarrow P'(x',y') = P'(y,x)$$

5. Reflection on the line y = -x

If the point P(x,y) is reflected against the line y = -x, then the image is obtained as follows:
Mapping $P(x,y) \rightarrow P'(x',y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is called the matrix corresponding to the reflection about the line y = -x. So, it can be written:
$M_{y=-x}: P(x,y) \rightarrow P'(x',y') = P'(-y,-x)$

6. Reflection on the line x = h

If the point P(x,y) is reflected against the line x = h, then the image is obtained as follows:
Mapping $P(x,y) \rightarrow P'(x',y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2h \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2h - x \\ y \end{pmatrix}$$

Matrix $\begin{pmatrix} 2h \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ is called the matrix corresponding to the reflection about the line x = h. So, it can be written:
$M_{x=h}: P(x,y) \rightarrow P'(x',y') = P'(2h - x, y)$

7. Reflection on the line y = k

If the point P(x,y) is reflected against the line y = k, then the image is obtained as follows:
Mapping $P(x,y) \rightarrow P'(x',y')$ can be determined by the matrix equation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 \\ 2k \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2k - y \end{pmatrix}$$

Matrix $\begin{pmatrix} 0 \\ 2k \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is called the matrix corresponding to the reflection about the line y = k. So, it can be written:
$M_{y=k}: P(x,y) \rightarrow P'(x',y') = P'(x, 2k - y)$

Discussion

1. Reflection on the X-axis
$M_x: P(x,y) \rightarrow P'(x',y') = P'(x,-y)$
If the point as follows is reflected on the X axis, so it can be written:
$A(2,2) \rightarrow A'(2,-2)$
$B(6,4) \rightarrow B'(6,-4)$
$C(2,6) \rightarrow C'(2,-6)$
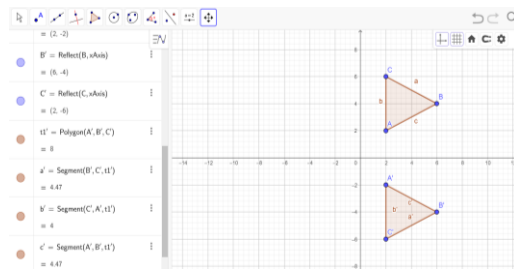By using Geogebra the following image is obtained:

Figure 2. Reflection on the X-axis.

2. Reflection on the Y-axis
$$M_y: P(x,y) \rightarrow P'(x',y') = P'(-x,y)$$
If the point as follows is reflected on the Y axis, so it can be written:
$A(4,6) \rightarrow A'(-4,6)$
$B(10,6) \rightarrow B'(-10,6)$
$C(10,2) \rightarrow C'(-10,2)$
$D(4,2) \rightarrow D'(-4,2)$
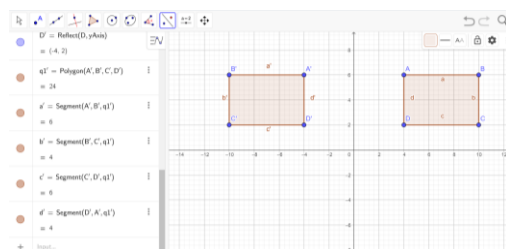By using Geogebra the following image is obtained:

Figure 3. Reflection on the Y-axis.

3. Reflection on the O (0,0)
$$M_o: P(x,y) \rightarrow P'(x',y') = P'(-x,-y)$$
If the point as follows is reflected on the O (0,0), so it can be written:
$B(10,5) \rightarrow B'(-10,-5)$
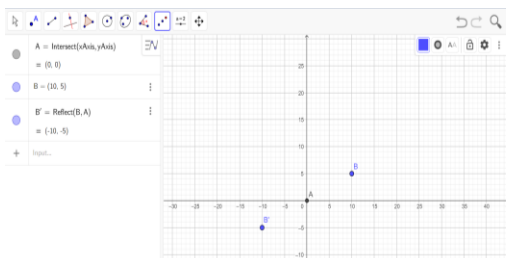By using Geogebra the following image is obtained:

Figure 4. Reflection on the O (0,0).

4. Reflection on the line y = x
$$M_{y=x}: P(x,y) \rightarrow P'(x',y') = P'(y,x)$$
If the point as follows is reflected on the line y = x, so it can be written:
$A(-10,10) \rightarrow A'(10,-10)$
$B(-5,10) \rightarrow B'(10,-5)$
$C(-5,5) \rightarrow C'(5,-5)$
$D(-10,5) \rightarrow D'(5,-10)$
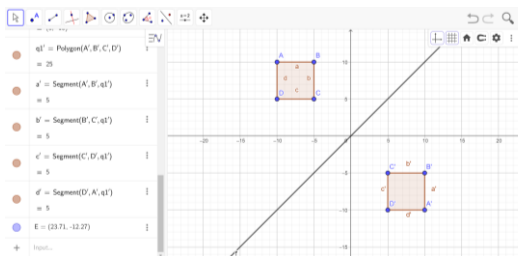By using Geogebra the following image is obtained:

Figure 5. Reflection on the line y=x.

5. Reflection on the line y = -x
$$M_{y=-x}: P(x,y) \rightarrow P'(x',y') = P'(-y,-x)$$
If the point as follows is reflected against the line y = -x, so it can be written:
$A(2,6) \rightarrow A'(-6,-2)$
$B(6,2) \rightarrow B'(-2,-6)$
$C(6,8) \rightarrow C'(-8,-6)$
By using Geogebra the following image is obtained:

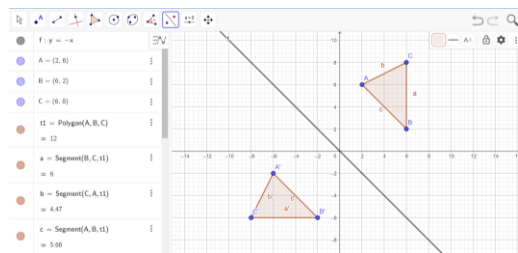*Figure 6.* Reflection on the line y = -x.

6. Reflection on the line x = h
$$M_{x=-h}: P(x,y) \rightarrow P'(x',y') = P'(2h-x,y)$$
If the point as follows is reflected against the line x = 2, so it can be written:
$A(8,6) \rightarrow A'(2.2-8,6) = (-4,6)$
$B(12,6) \rightarrow B'(2.2-12,6) = (-8,6)$
$C(14,4) \rightarrow C'(2.2-14,4) = (-10,4)$
$D(6,4) \rightarrow D'(2.2-6,4) = (-2,4)$
By using Geogebra the following image is obtained:

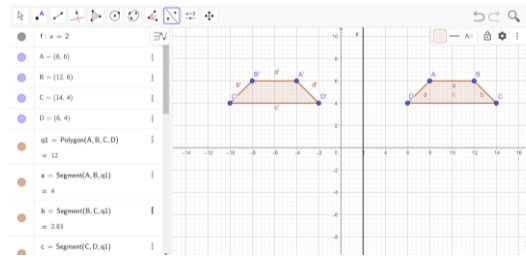*Figure 7.* Reflection on the line x = 2.

7. Reflection on the line y = k
$$M_{y=k}: P(x,y) \rightarrow P'(x',y') = P'(x,2k-y)$$
If the point as follows is reflected against the line y = 2, so it can be written:
$A(5,-5) \rightarrow A'(5,2.2+5) = (5,9)$
$B(15,-5) \rightarrow B'(15,2.2+5) = (15,9)$
$C(20,-15) \rightarrow C'(20,2.2+15) = (20,19)$
$D(10,-15) \rightarrow D'(10,2.2+15) = (10,19)$
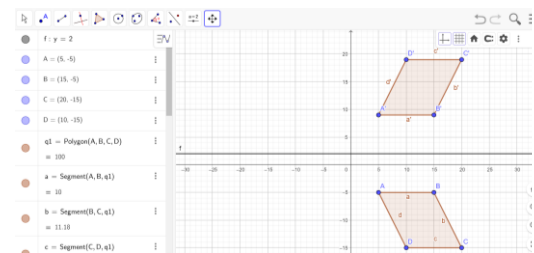By using Geogebra the following image is obtained:

Figure 8. Reflection on the line y = 2.

Conclusion

The conclusions of this study are:
1. Using Geogebra software can help visualize points and planes that are reflected at points O (0,0), on the X, Y axes, lines y = x, y = -x, x = h and x = k
2. By using the Geogebra software, it can be seen that the results of point and plane reflections are reflected at point O (0,0), about the X, Y axes, the lines y = x, y = -x, x = h and x = k.
3. Reflection on the X-axis $\rightarrow M_x: P(x,y) \rightarrow P'(x',y') = P'(x,-y)$
4. Reflection on the Y axis $\rightarrow M_y: P(x,y) \rightarrow P'(x',y') = P'(-x,y)$
5. Reflection on the O (0,0) $\rightarrow M_o: P(x,y) \rightarrow P'(x',y') = P'(-x,-y)$
6. Reflection on the line y = x $\rightarrow M_{y=x}: P(x,y) \rightarrow P'(x',y') = P'(y,x)$
7. Reflection on the line y = -x

$\rightarrow M_{y=-x}: P(x,y) \rightarrow P'(x',y') = P'(-y,-x)$

8. Reflection on the line x = h

$\rightarrow M_{x=h}: P(x,y) \rightarrow P'(x',y') = P'(2h-x,y)$

9. Reflection on the line y = k

$\rightarrow M_{y=k}: P(x,y) \rightarrow P'(x',y') = P'(x,2k-y)$

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